DIOPHANTINE ANALYSIS.

55. Proposed by O. W. ANTHONY, M. Sc. Instructor in Mathematics in Boys' High School, New York City.

Construct a general Magic Square whose sum is $3m$.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

$$\begin{array}{lll} m+bn & m-(a+b)n & m+an \\ m+(a-b)n & m & m-(a-b)n \\ m-an & m+(a+b)n & m-bn \end{array}$$

II. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

I assume that the method of constructing Magic Squares is understood, and that they are made up of numbers in arithmetical progression.

Let p be the first term, q the common difference, and n the number of rows.

Then n^2=number of cells and of the numbers used; the sum of the series divided by n gives the sum of the numbers in each row=$3m$.

Or $\dfrac{n^2[r(n^2-1)+2p]}{2n}=3m$, or, $n[q(n^2-1)+2p)=6m$............ ......(1).

As there are three unknown quantities, two of them may be assumed, and there will be a result for each assumption of both numbers.

Then $n=3$ and reducing we have $4q+p=m$; or $p=m-4q$, in which m may be any multiple of q greater than $4q$.

Take $m=5q$, then $q=m/5$, and $p=q$. In this m may be any number divisible by 5, and for every value we have a magic square, whose sum is $3m$.

Again, take $m=6q$, then $p=2q$, and $q=m/6$; m may be any number divisible by 6; and for every value we have a magic square, whose sum is $3m$.

In the same manner, we find that m may be any number divisible by any one of the numbers in the natural series from 5 upwards.

Now take $n=5$; substitute in (1), and we have by reducing $120q+10p=6m$. To simplify assume $m=5t$, and we have $12q+p=3t$; or $p=3t-12q$, in which t may be any number greater than $4q$.

Take $t=5q$; then $m=25q$, and $p=3q$, and $q=m/25$. So m may be any number divisible by 25.

In same manner, take $t=6q$, $7q$, etc., and we have $q=m/30$, $m/35$, etc., so that we may have m=any number divisible by 5 times any one of the natural numbers from 5 upwards, and as many magic squares of 5 rows.

In the same manner, we may obtain similar results by taking n=any other odd number.

Or, we can obtain a general solution directly from (1). Take $m=nt$, $n=2s+1$, substitute and reduce and we find $p=3t-2q(s^2+s)$, in which t may be $=q(s^2+s)$, or any multiple of it. Take $t=q(s^2+s)$; and then $p=q(s^2+s)$, (the first term); and $m=nt=q(2s+1)(s^2+s)$, and $q=m/[s(s+1)(2s+1)]$ (the common difference); in which s may be any integral, and $m=s(s+1)(2s+1)$, or any multiple of it.

But this formula does not enable us to obtain least values of p, q, and m, as n varies.

56. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

If $\phi(R)$ is the number of integers which are less than R and prime to it, and if y is prime to R, show that $y^{\Phi(R)}-1\equiv0(\text{mod}.R)$.

Solution by the PROPOSER, and J. O. MAHONEY, B. E., M. Sc., Lynnville, Tenn.

Let $1, m, n, p, \ldots\ldots(R-1)$ denote the $\phi(R)$ numbers less than R and prime to it ; now y can be any one of those numbers.

$\therefore y, my, ny, py, \ldots\ldots(R-1)y$ are all prime to R and all different.

There are $\phi(R)$ of such products and since when these products are divided by R the remainders are all prime to R and all different, the $\phi(R)$ remainders must be $1, m, n, p, \ldots\ldots(R-1)$ though not necessarily in this order.

$\therefore y.my.ny.py\ldots\ldots(R-1)y$ must differ from $1.m.n.p\ldots\ldots(R-1)$ by a multiple of R.

$\therefore \{y^{\Phi(R)}-1\}mnp\ldots\ldots(R-1)=$a multiple of R.

But $mnp\ldots\ldots(R-1)$ is prime to R.

$\therefore y^{\Phi(R)}-1\equiv0(\text{mod}.R)$.

57. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

Each of *five* of the digits may be the terminal figure of a perfect integral square. Each of *eighteen* combinations of two digits may be the *two* terminal figures of an integral square. Each of *one hundred and nineteen* combinations of three digits may be the *three* terminal figures of an integral square. *Under these conditions*, what is the greatest number of arrangements of the nine digits, all taken together, whose three terminal figures shall be those of a square number ?

No solution of this difficult problem has been received. Can any of our readers furnish the desired solution ? EDITOR.

MISCELLANEOUS.

53. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics in Curry University, Pittsburg, Pa.

(a) What is the highest north latitude in which the Sun will shine in at the north window of a building at least once in a year ?

(b) How many days will it shine in at the north window of a building in latitude 41° N. ?

Note by SAMUEL HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

Whenever the Sun, or any part of it, is north of the prime vertical, it must then shine on the north side of buildings. From the time of vernal equinox, to the autumnal equinox, the Sun will be north of the prime vertical during some part of every day, and will shine on the north side of buildings some part of *every* day for about half a year, and in *all* latitudes north of the equator. Hence the answer for (a) is 90° N. latitude, and for (b) 186 days, but if the Sun's upper